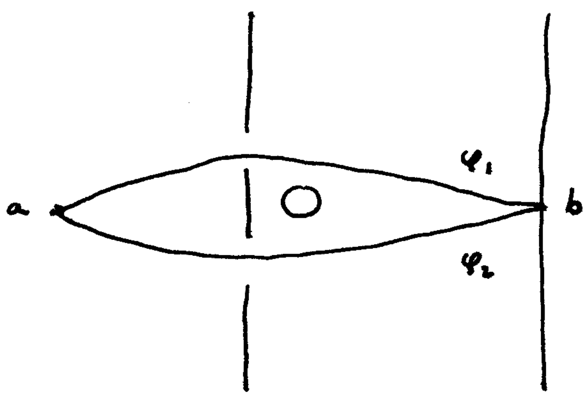
DOUBLE SLIT PATTERN SHIFTS EVEN THOUGH $\vec{E}=0$ $\vec{B}=0$

SHIFT DEPENDS ON $\vec{A}$

$\therefore$ $\vec{A}$ IS REAL $\vec{E}$ AND $\vec{B}$ ARE NOT




WHEN $\vec{A}=0$ $\qquad \varphi_1 = e^{iS_1/\hbar}$

$$\varphi_2 = e^{iS_2/\hbar}$$

WHEN $\vec{A}\neq 0$ $\qquad \varphi_1 = e^{iS_1'/\hbar}$

$$S' = S + \frac{e}{c}\int \vec{A}\cdot d\vec{l}$$

$L' = L + \Delta L$

$$\Delta\varphi = \Delta\varphi_1 - \Delta\varphi_2$$

$$= \frac{e}{\hbar c}\int \vec{A}_1 \cdot d\vec{l} - \frac{e}{\hbar c}\int \vec{A}_2 \cdot d\vec{l}$$

$$= \frac{e}{\hbar c}\oint \vec{A}\cdot d\vec{l} = \frac{e}{\hbar c}\int (\vec{\nabla}\times\vec{A})\cdot d\vec{S}$$

$$= \frac{e}{c}\int \vec{B}\cdot d\vec{S} = \frac{e}{\hbar c}\,\Phi_B$$

EXISTENCE OF AB DEPENDS ON $\vec{A} \neq 0$ IN REGION $\vec{B} = 0$ $\vec{E} = 0$

SIZE OF AB DEPENDS ON $\Phi_B$

PERIOD $\quad \Phi_0 = \dfrac{hc}{e} = 4.14 \times 10^{-7}$ gauss cm$^2$

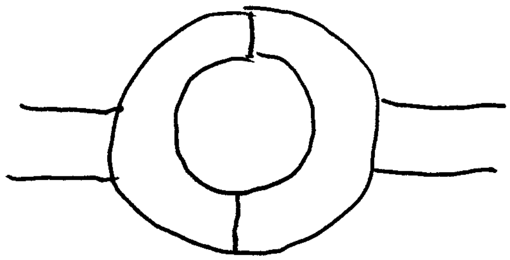
MAGNETIC FLUX QUANTUM

WHEN TIME DEPENDENT

$$\Delta\varphi = -\frac{e}{\hbar c}\int V(t)\, dt$$

AHARONOV CASHER

JOSEPHSON EFFECT III CH 21



INTERFERENCE OF SUPERCURRENTS

NANOSCOPIC MESOSCOPIC RINGS

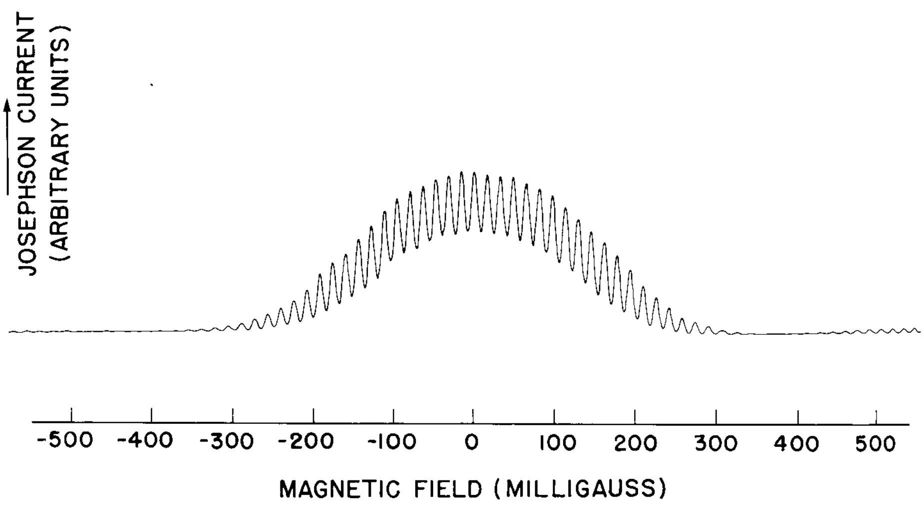



Fig. 21–8. A recording of the current through a pair of Josephson junctions as a function of the magnetic field in the region between the two junctions (see Fig. 21–7). [This recording was provided by R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau of the Scientific Laboratory, Ford Motor Company.]

The Josephson current through a double junction was recently measured[17] as a function of the magnetic field in the area between the junctions. The results are shown in Fig. 21–8. There is a general background of current from various effects we have neglected, but the rapid oscillations of the current with changes in the magnetic field are due to the interference term $\cos q_e\Phi/\hbar$ of Eq. (21.52).

One of the intriguing questions about quantum mechanics is the question of whether the vector potential exists in a place where there's no field.[18] This experiment I have just described has also been done with a tiny solenoid between the two junctions so that the only significant magnetic $\boldsymbol{B}$ field is inside the solenoid and a negligible amount is on the superconducting wires themselves. Yet it is reported that the amount of current depends oscillatorily on the flux of magnetic field inside that solenoid even though that field never touches the wires—another demonstration of the "physical reality" of the vector potential.[19]

I don't know what will come next. But look what can be done. First, notice that the interference between two junctions can be used to make a sensitive magnetometer. If a pair of junctions is made with an enclosed area of, say, 1 mm$^2$, the maxima in the curve of Fig. 21–8 would be separated by $2 \times 10^{-6}$ gauss. It is certainly possible to tell when you are 1/10 of the way between two peaks; so it should be possible to use such a junction to measure magnetic fields as small as $2 \times 10^{-7}$ gauss—or to measure larger fields to such a precision. One should be able to go even farther. Suppose for example we put a set of 10 or 20 junctions close together and equally spaced. Then we can have the interference between 10 or 20 slits and as we change the magnetic field we will get very sharp maxima and minima. Instead of a 2-slit interference we can have a 20- or perhaps even a 100-slit interferometer for measuring the magnetic field. Perhaps we can predict that the measurement of magnetic fields will—by using the effects of quantum-mechanical interference—eventually become almost as precise as the measurement of wavelength of light.

These then are some illustrations of things that are happening in modern times—the transistor, the laser, and now these junctions, whose ultimate practical applications are still not known. The quantum mechanics which was discovered in 1926 has had nearly 40 years of development, and rather suddenly it has begun to be exploited in many practical and real ways. We are really getting control of nature on a very delicate and beautiful level.

I am sorry to say, gentlemen, that to participate in this adventure it is absolutely imperative that you learn quantum mechanics as soon as possible. It was our hope that in this course we would find a way to make comprehensible to you at the earliest possible moment the mysteries of this part of physics.

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[17] Jaklevic, Lambe, Silver, and Mercereau, *Phys. Rev. Letters* **12**, 159 (1964).
[18] Jaklevic, Lambe, Silva, and Mercereau, *Phys. Rev. Letters* **12**, 274 (1964).
[19] See Volume II, Chapter 15, Section 15–5.

## *Feynman's Epilogue*

Well, I've been talking to you for two years and now I'm going to quit. In some ways I would like to apologize, and other ways not. I hope—in fact, I know—that two or three dozen of you have been able to follow everything with great excitement, and have had a good time with it. But I also know that "the powers of instruction are of very little efficacy except in those happy circumstances in which they are practically superfluous." So, for the two or three dozen who have understood everything, may I say I have done nothing but shown you the things. For the others, if I have made you hate the subject, I'm sorry. I never taught elementary physics before, and I apologize. I just hope that I haven't caused a serious trouble to you, and that you do not leave this exciting business. I hope that someone else can teach it to you in a way that doesn't give you indigestion, and that you will find someday that, after all, it isn't as horrible as it looks.

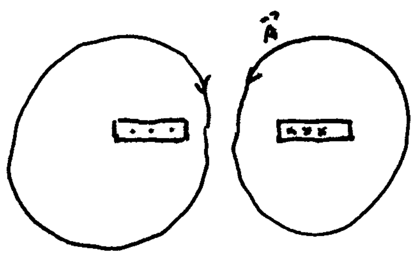
Finally, may I add that the main purpose of my teaching has not been to prepare you for some examination—it was not even to prepare you to serve industry or the military. I wanted most to give you some appreciation of the wonderful world and the physicist's way of looking at it, which, I believe, is a major part of the true culture of modern times. (There are probably professors of other subjects who would object, but I believe that they are completely wrong.)

Perhaps you will not only have some appreciation of this culture; it is even possible that you may want to join in the greatest adventure that the human mind has ever begun.

TONOMURA EXPERIMENT

PREVENT $\vec{B}$ LEAKAGE

TOROID




SUPERCONDUCTING SHIELD

KEEP $e^-$ OUT OF SAMPLE

$\Rightarrow$ THICK SUPERCONDUCTING SHIELD

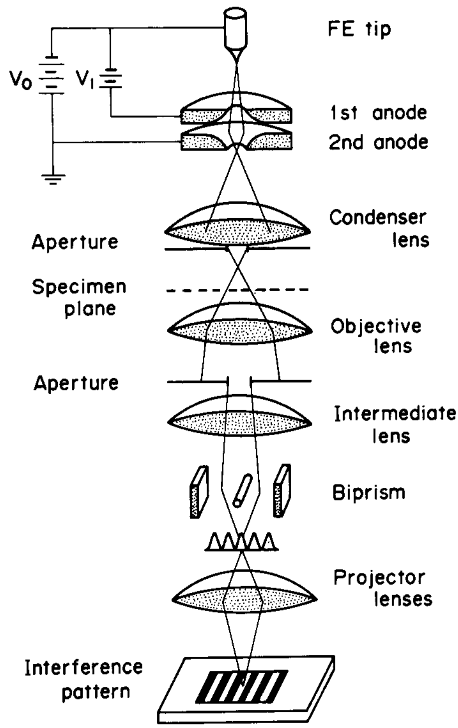



**Figure 1.1.** Electron interference with a field-emission electron microscope. Wave front splitting occurs at the biprism. (Courtesy of A. Tonomura, Hitachi ARL.)

(effectively the two slits of the apparatus), the two components of the electron beam recombined in the observation plane of the microscope where the build-up of a pattern of interference fringes was recorded on film and on a TV monitor.

The appearance of a fringe pattern is not in itself extraordinary. After all, if electrons were actually waves, then the experimental configuration would represent a type of wavefront-splitting interferometer, and there is nothing unusual about the linear superposition of waves to generate an interference pattern. What is startling, however, is the observed *emergence* of the fringe pattern in a microscope of approximately 1.5 m length under conditions where the mean interval between successive electrons is over 100 km! Clearly a given electron had been detected long before the succeeding electron was "born" at the field-emission tip. Under these circumstances it is unlikely that there can be any sort of cooperative interaction between the electrons of the beam.

Electron detection events appear on the TV monitor one by one at random locations as illustrated in Figure 1.2. The first few hundred scattered spots hardly hint at any organization. However, by the time some hundred thousand electrons have been recorded, stark alternating stripes of white and black stand out sharply as if made by the two-slit interference of laser light.

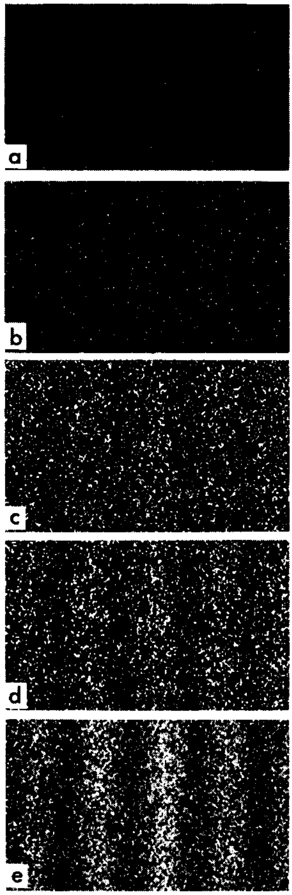



**Figure 1.2.** Evolution of electron interference pattern in time. Electrons arrive at the rate of approximately 1000 per second. The number recorded in each frame is: (a) 10; (b) 100; (c) 3000; (d) 20,000; and (e) 70,000. (Courtesy of A. Tonomura, Hitachi ARL.)

Indeed, except for the marked difference in wavelengths—about 500 nm for visible light and 0.005 nm (a tenth the diameter of a hydrogen atom) for the 50 kV electrons—the uninformed observer could not tell whether the fringe pattern was created by light or by particles. If the location of each electron arrival is random, and there is no communication between electrons, how then can the overall spatial distribution of detected electrons manifest a coherently organized pattern? *That* is the enigma of quantum mechanics to which Feynman referred.

Since the two-slit electron interference experiment is conceptually the simplest, if archetypal, example of quantum interference, it is worth examining it quantitatively in more detail, if only to introduce geometric and dynamical quantities that will be encountered again later. The speed $\beta$ (relative to that

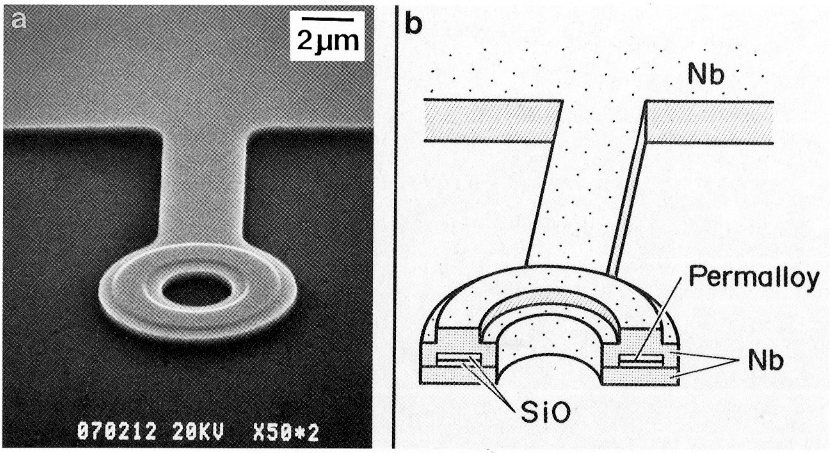



**Fig.6.14.** Toroidal ferromagnet covered with a superconductor: (**a**) Scanning electron micrograph, and (**b**) cross-sectional diagram. The bridge between the toroidal sample and the Nb plate is for cooling the sample

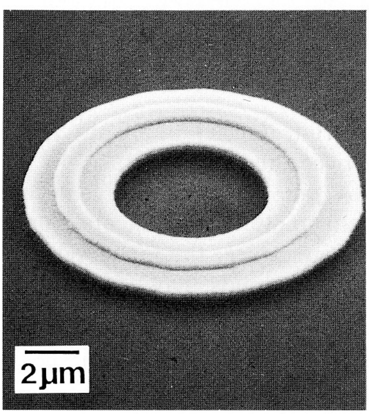



**Fig.6.15.** Scanning electron micrograph of a toroidal ferromagnet without a bridge

heat conduction between the sample and the low-temperature specimen stage. When isolated toroidal samples were placed on a carbon film, as shown in Fig.6.15, they could not be cooled below the $T_c$ of the covering superconductor.

In preparing the samples, special attention was paid to attaining perfect contact between the two niobium layers, because even a slight oxidation layer between them would break down the superconducting contact and allow the magnetic field to leak out. The niobium oxide produced on top of the lower niobium layer in the lithography processes had therefore to be removed by ion sputtering before the upper niobium layer was laid cannot penetrate the magnet, a small amount of leakage flux from the magnet might influence the electron beam. To prevent this possibility, the shielding metal layer was made of superconducting material. The Meissner effect prevents magnetic fields from passing through a superconducting layer. Consequently, electron holography showed that no magnetic fields leaked outside the fabricated toroidal sample.

It thus became possible to test for the AB effect by using this sample under conditions where there was no overlap between the magnetic field and an electron beam. A tiny toroidal magnet less than 10 $\mu$m in diameter had to be completely covered by a superconductor, without even the slightest gap. The thickness of the superconducting layer had to be greater than the penetration depth. Such samples could actually be fabricated by using advanced photolithographic techniques. The fabrication process is complicated, so only its principle aspects are represented in Fig.6.13.

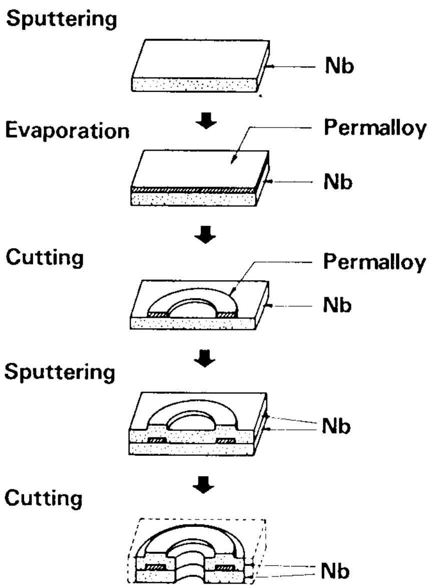



**Fig.6.13.** Photolithographic process for fabricating a toroidal ferromagnet covered with a superconductor

A 20-nm thick permalloy film was prepared by vacuum evaporation on a silicon wafer covered with a 250-nm thick niobium film. After the toroidal shape was cut out of the permalloy film, a 300 nm niobium film was sputtered on its surface. Then the toroidal sample was cut so that the permalloy toroid was completely covered with the niobium layer. Finally a copper film 50 to 200 nm thick was evaporated on all of its surfaces; this film prevented electron-beam penetration into the magnet and kept the sample from experiencing charge-up and contact-potential effects. In a scanning electron micrograph of the sample (Fig.6.14), one can see a bridge between the toroid and a niobium plate. This bridge ensures good

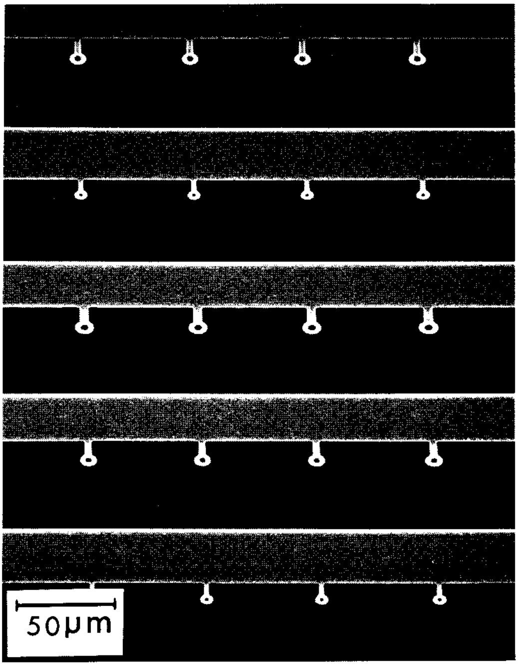



Fig.6.16. Array of toroidal samples

down. In another experiment, a maximum current density of 40 mA/$\mu$m$^2$ was confirmed to pass through the contact between the two layers at 4.2 K. This current was much larger than the 10 mA/$\mu$m$^2$ persistent current calculated to be necessary for quantizing the magnetic flux. Except for a slight flux change due to changes in sample temperature, the magnetic flux of this sample cannot change very much. Various toroids, having different flux values, were therefore connected to the niobium plate (Fig.6.16).

**b) Experimental Results**

Experiments were carried out in a manner similar to that described in Sect.6.5.1. An electron hologram of a sample was first formed by using a 150 kV field-emission electron microscope, and then the relative phase shift for the sample was optically reconstructed using a HeNe laser. A preliminary experiment was carried out to test whether there would be any observable interaction between an electron beam and a simple toroidal superconductor containing no magnet. This experiment was to confirm that the phase of an electron beam passing near a superconductor is not influenced by it, even though the Meissner effect prevents the magnetic field accompanying the electron beam from passing through the superconductor.

This preliminary experiment measured the phase difference between electron beams passing inside and outside of the 300 nm thick niobium toroid which was completely covered by a layer of 50 to 200 nm thick

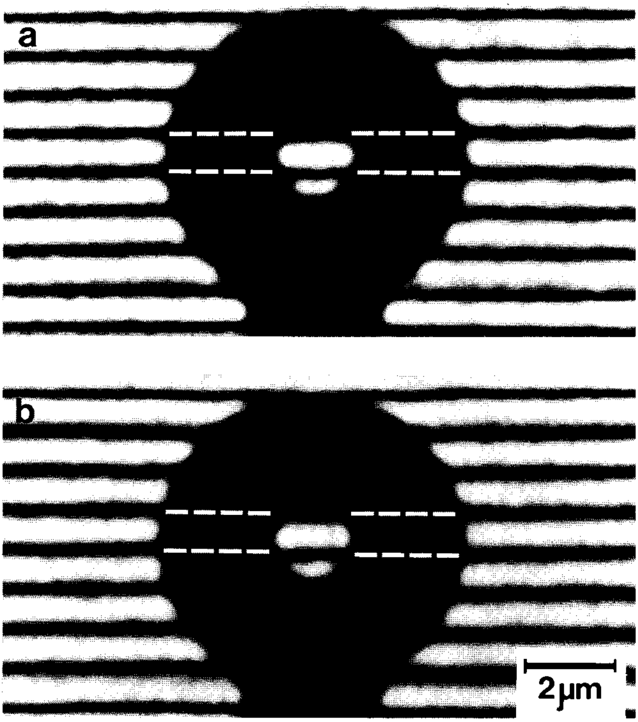



Fig.6.17. Experimental results showing no interaction between an electron beam and superconducting toroid without a magnet inside: (a) Normal state at 15 K, and (b) superconducting state at 4.5 K

copper. When the temperature of the toroid was varied above and below its critical temperature (9.2K), the twofold phase-amplified interference micrographs showed no phase difference in the normal or the superconducting state (Fig.6.17). Another experiment confirmed that niobium samples actually became superconducting at 4.5 K in this experiment: an external magnetic field was applied to a toroidal sample and the flux trapped in the sample hole at 4.5 K was observed by electron holography. These preliminary experiments confirmed that the toroidal superconductor itself would not influence an incident electron beam, that is, it would not produce a relative phase shift between a beam passing through the hole and a beam passing outside the toroid.

Fabricated toroidal samples having no leakage flux were then selected by using electron-holographic interferometry, and the magnetic flux leaking from the samples was measured. This experiment confirmed that there was no interaction, to within the precision of a $2\pi/10$ phase shift, between the electron beam and the superconducting toroid. The samples were not always free of leakage flux (Fig.6.18). Leakage was found to be appre-

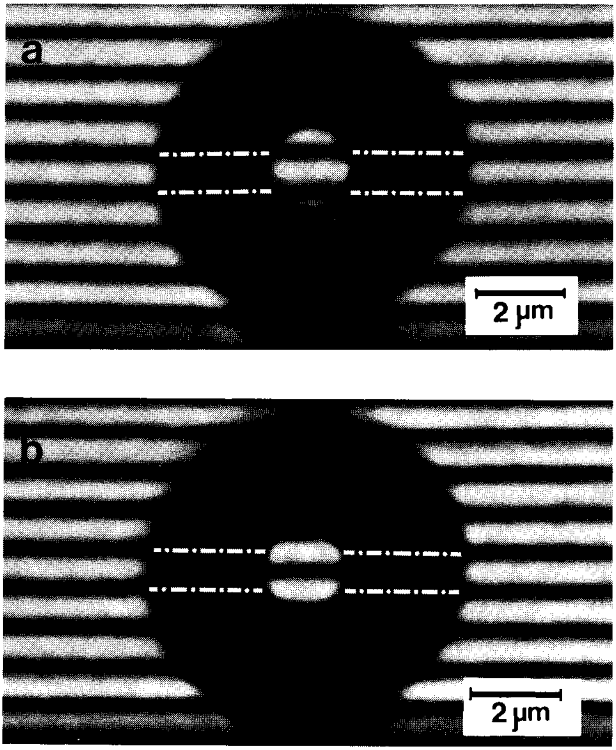



**Fig.6.19.** Experimental results showing the Aharonov-Bohm effect: (**a**) Phase shift of 0, and (**b**) phase shift of $\pi$

No quantization was observed in samples having even a slightly oxidized layer between the two niobium layers. The occurrence of flux quantization can therefore be taken as assurance that the niobium layer actually becomes superconducting, that the superconductor completely surrounds the magnetic flux, and that the Meissner effect prevents any flux from leaking outside. In Fig.6.20b the phase shift just above $T_c$ was $0.8\pi$ and was closer to $\pi$ than to 0, resulting in a jump to $\pi$.

In this way, the existence of the AB effect was confirmed without any doubt.

STAT MECH WITH PATH INTEGRALS

$$e^{iS/\hbar}$$

$$S = \int L \, dt$$

$t \to -i\, t_E$ EUCLIDEAN TIME

$$ds^2 = dt^2 + dx^2 + dy^2 + dz^2$$

IMAGINARY TIME

MINKOWSKI TIME

length squared $= ds^2 = dt^2 - dx^2 - dy^2 - dz^2$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$e^{iS/\hbar}$

VIOLENT RAPID OSCILLATIONS

AVERAGE TO ZERO

HARD TO CALCULATE

$e^{-S_E/\hbar}$

OSC $\rightarrow$ DECAYING EXP

EASY TO CALCULATE

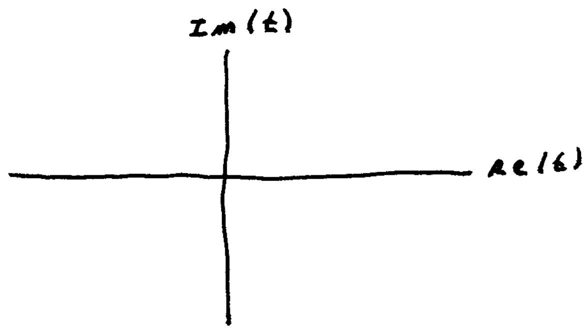
THEN TRANSFORM BACK

LIKE FOURIER TRANSFORMS : LAPLACE TRANSFORMS

$$g(\omega) = \int e^{-i\omega t} f(t)\, dt$$

$$g(s) = \int e^{-st} f(t)\, dt$$

COMPLEX TIME




WICK ROTATION

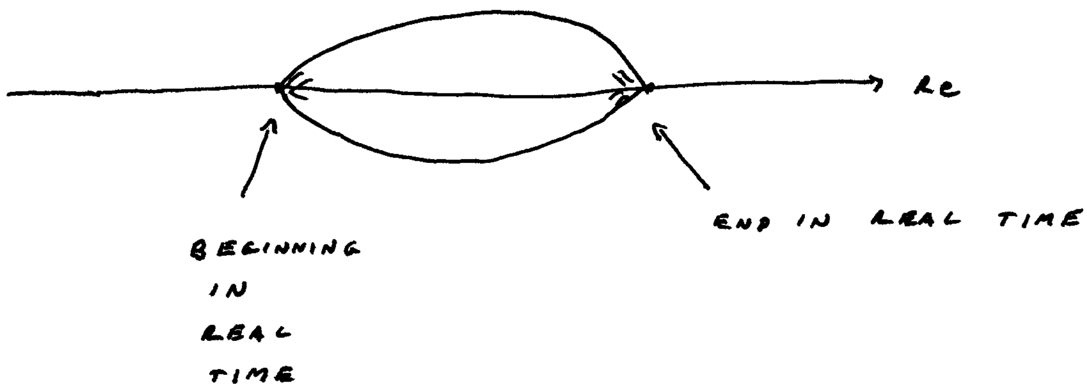
| | | |
|---|---|---|
| CLASSICAL | | EUCLIDEAN |
| STAT MECH | $\Longleftrightarrow$ | QFT |
| N-dim SPACE | | N-dim SPACETIME |

| | | |
|---|---|---|
| | | FINITE TEMP |
| QUANTUM | | EUCLIDEAN |
| STAT MECH | $\Longleftrightarrow$ | QFT |
| N-dim SPACE | | (N+1)-dim SPACETIME |

$$e^{-iHt} \qquad\qquad e^{-\beta H}$$

TEMPERATURE IS EQUIVALENT TO CYCLIC IMAGINARY TIME

IMAGINARY TIME IN COSMOLOGY




NO BEGINNING

NO END

IN COMPLEX TIME

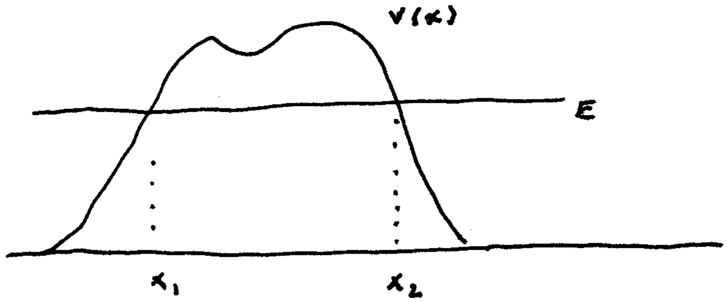
NO BOUNDARY

STUDENT: ONE BOUNDARY

FINITE IN SPACE AND TIME:

W/ NO BOUNDARY

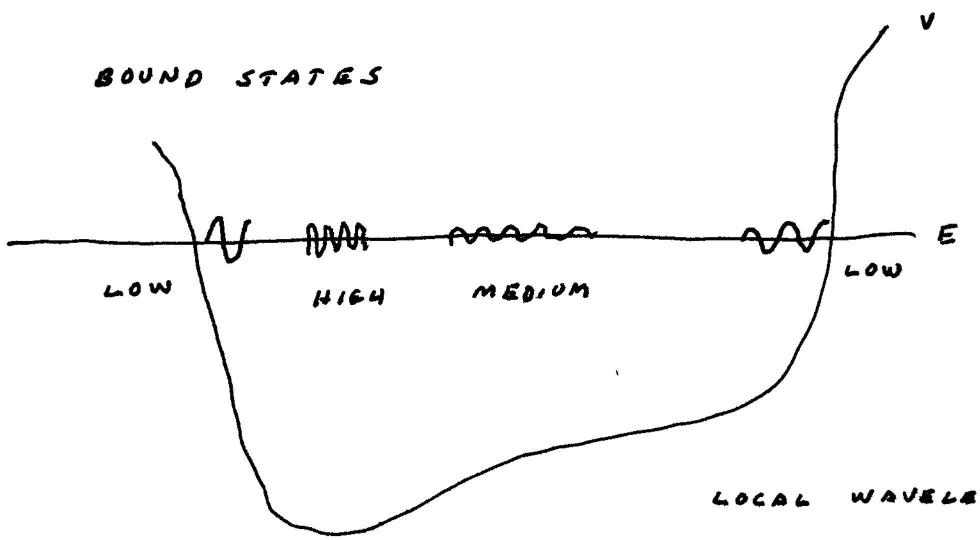
PATH INTEGRAL ALONG CLASSICAL PATH ⇒ WKB APPROXIMATION




$$T(E) \doteq e^{-\text{ACTION UNDER BARRIER}}$$

$$= e^{-S/\hbar}$$

BOUND STATES




$$\frac{\hbar^2 k^2}{2m} = E - V$$

LOCAL WAVELENGTH

WAVE FCN OSC "FAST"
COMPARED TO CHANGES
IN $V(x)$

$$k = \sqrt{\frac{2m}{\hbar^2}\left(E - V(x)\right)}$$

$k(x)$

SEMI CLASSICAL LIMIT

# LECTURE 10: THE WKB METHOD

WENTZEL
KRAMERS
BRILLOUIN

INVENTED SIMULTANEOUSLY IN 1926 FOR QM PROBLEMS, BUT GENERAL MATHEMATICAL TECHNIQUE KNOWN EARLIER BY ~~JEFFRIES AND~~ LIOUVILLE (1837) ~~FOR MATH PROBLEMS~~ RAYLEIGH (1912) and JEFFRIES (1923)

HOLLAND: KWB

FRANCE: BWK

ENGLAND: JWKB JEFFRIES

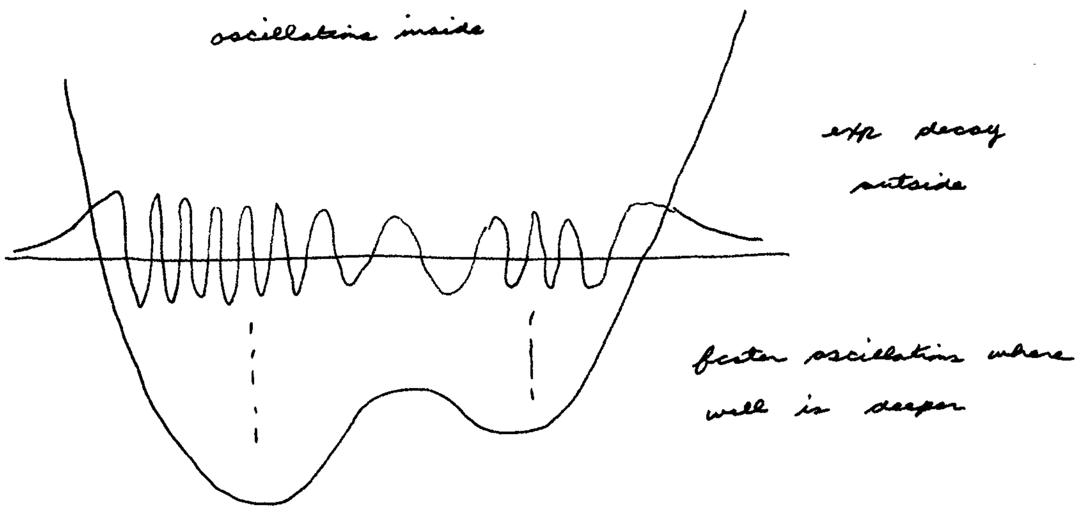
USSR: QUASI CLASSICAL Landau-Lifshitz

a.k.a. SEMI CLASSICAL METHOD, CLASSICAL APPROXIMATION, PHASE INTEGRAL METHOD

TWO MAJOR APPLICATIONS:

(1) BOUND STATES

SUPPOSE WE WANT TO FIND THE BOUND STATES OF A ~~[illegible]~~ "SOPHISTICATED" POTENTIAL, LIKE




local spatial wavelength $\qquad \dfrac{\hbar^2 k^2}{2m} = E - V \qquad k = \sqrt{\dfrac{2m}{\hbar^2}(E-V)}$

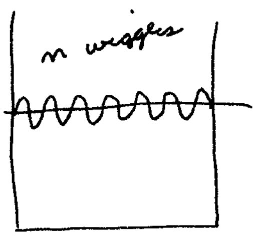
NOW WE HAVE THE COMPLETE WAVEFCN FOR ANY E ... BUT HOW DO WE FIND THE BOUND STATES?

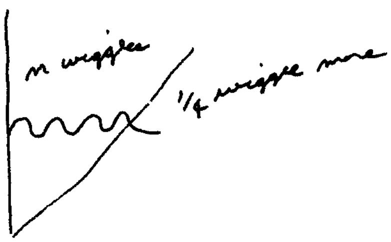
WKB QUANTIZATION CONDITIONS: $n = 1, 2, 3, \ldots$

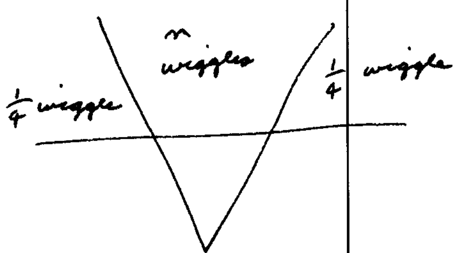
★ $$\int_{x_1}^{x_2} p(x')\, dx' = n\pi\hbar$$ TWO INFINITE WALLS

$$= \left(n - 1/4\right)\pi\hbar$$ ONE INFINITE WALL, ONE PENETRABLE WALL

$$= \left(n - 1/2\right)\pi\hbar$$ TWO PENETRABLE WALLS



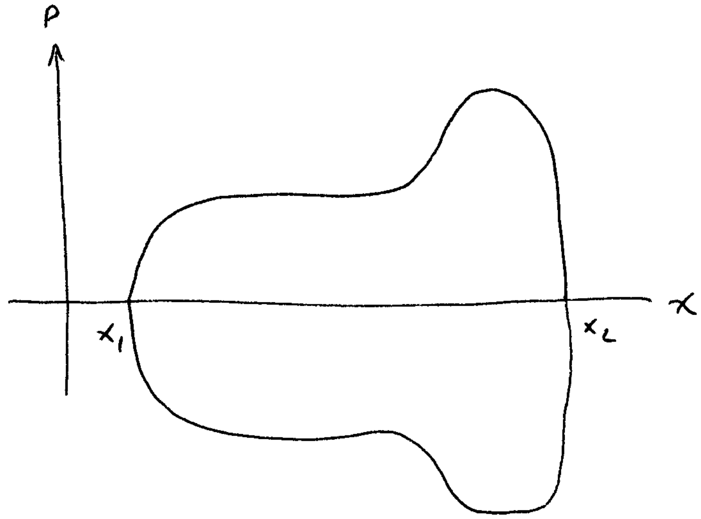
APPLICATION

1) FIND CLASSICAL TURNING POINTS

2) DO THE INTEGRAL

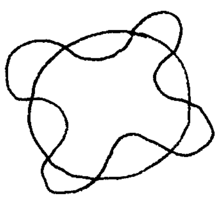
3) SET THE RESULT EQUAL TO THE STATE YOU SEEK, AND SOLVE FOR THE ENERGY

N.B. This is intimately related to the old BOHR-SOMMERFELD QUANTIZATION CONDITION




$$\oint \text{ACTION} = n\hbar = \int_{x_1}^{x_2} p(x')\,dx' + \int_{x_2}^{x_1} -p(x')\,dx'$$

and to BOHR'S SIMPLE PICTURE



$$\oint \frac{dx}{\lambda} = n$$

$$\int_{x_1}^{x_2} p(x)\,dx = (n+1/2)\pi \frac{h}{2\pi}$$

generalize this

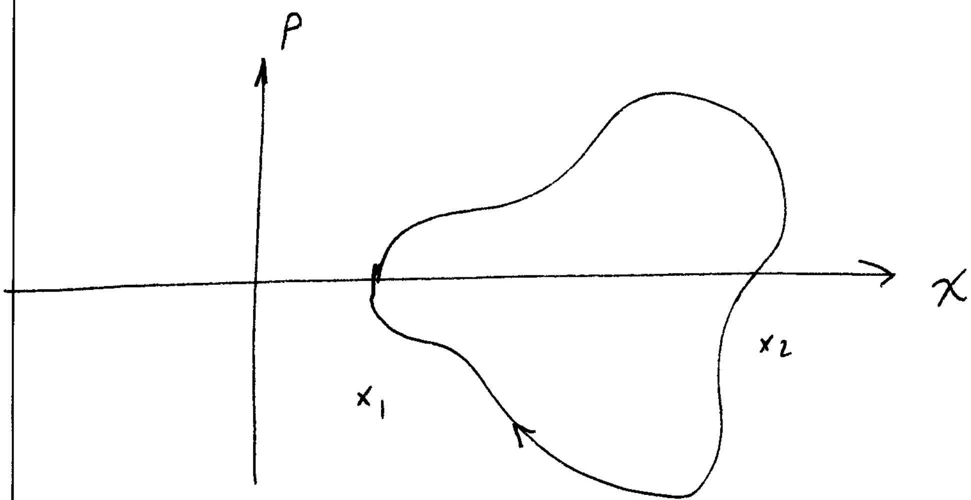
$$\oint p(x)\,dx = 2\int_{x_1}^{x_2} p(x)\,dx = \cancel{(n+1/2)2\pi\hbar} = 2(n+1/2)\frac{h}{2}$$

$$= (n+1/2)h$$

where $A = A'$ if $n$ is even

$A = -A'$ if $n$ is odd.

compare with
classical motion in phase space



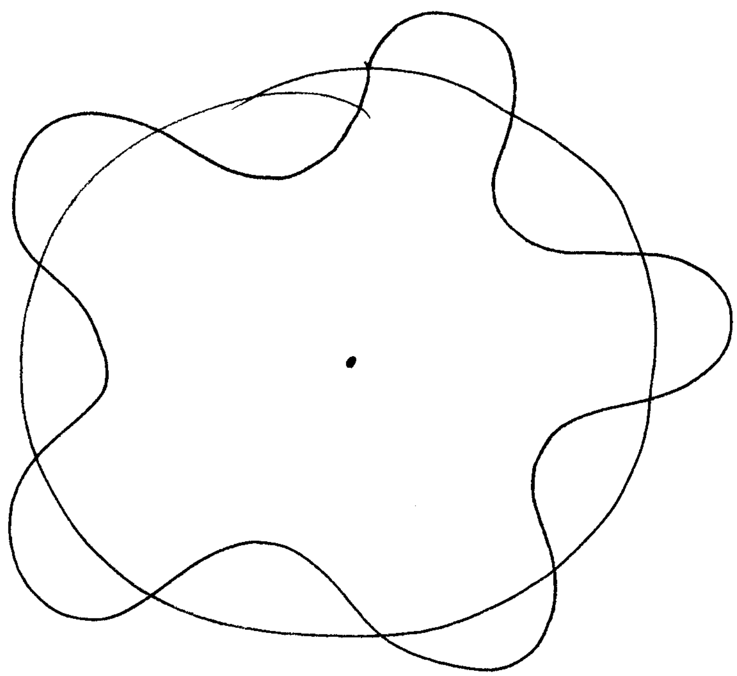

$$\oint p(x)\,dx = \text{area} = (n+1/2)h$$

very similar to BOHR-SOMMERFELD QUANTIZATION

$$\text{RULE} \qquad \oint \overset{\text{ACTION}}{\cancel{p(\theta)\,d\theta}} = \overset{\oint p(x)\,dx\,=}{\cancel{(n+1/2)\hbar}}\; n h$$

$\cancel{p(x)\,dx}$

PICTORIALLY:



Stationary states have integral number of wavelengths. $\lambda = \frac{h}{p}$

$$\oint \frac{dx}{\lambda} = n + 1/2$$

a half integer number of wavelengths have leaked into the classically forbidden region!

sharp potential $\psi = 0$ at TP $\Rightarrow$ no leakage.

INSTANTONS

WKB IN EUCLIDEAN TIME

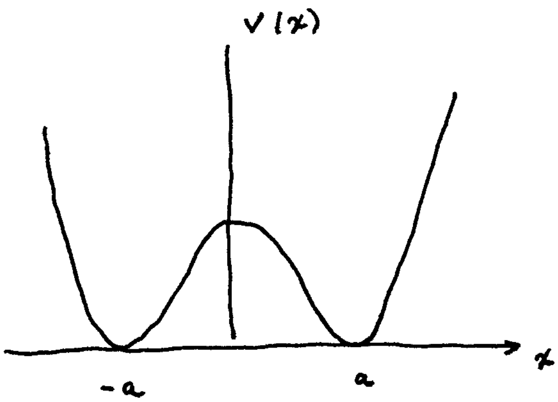
WORD "INSTANTON" INVENTED BY t' HOOFT

"ON" SIMILAR TO SOLITON

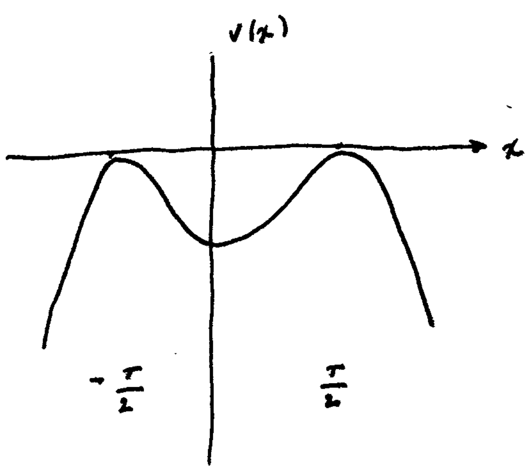
"INSTANT" STRUCTURE IN TIME

PHONON, PLASMON $\Big\}$ QUASI-PARTICLES

POLYAKOV PSEUDO-PARTICLES



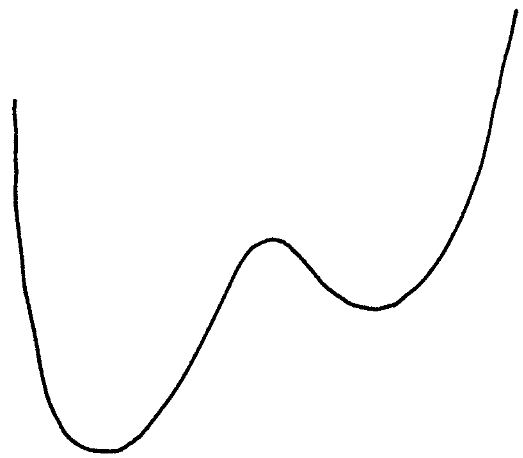

QUANTUM TUNNELING PROBLEM IN SPACETIME (QFT)




EUCLIDEAN FT

let $T \to \infty$

FALSE VACUUM



# 7

# The uses of instantons
(1977)

## 1 Introduction

In the last two years there have been astonishing developments in quantum field theory. We have obtained control over problems previously believed to be of insuperable difficulty and we have obtained deep and surprising (at least to me) insights into the structure of the leading candidate for the field theory of the strong interactions, quantum chromodynamics. These goodies have come from a family of computational methods that are the subject of these lectures.

These methods are all based on semiclassical approximations, and, before I can go further, I must tell you what this means in the context of quantum field theory.

To be definite, let us consider the theory of a single scalar field in four-dimensional Minkowski space, with dynamics defined by the Lagrangian density

$$\mathscr{L}=\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{2}m^2\phi^2-g^2\phi^4. \tag{1.1}$$

For classical physics, $g$ is an irrelevant parameter. The easiest way to see this is to define

$$\phi'=g\phi. \tag{1.2}$$

In terms of $\phi'$,

$$\mathscr{L}=\frac{1}{g^2}(\tfrac{1}{2}\partial_\mu\phi'\partial^\mu\phi'-\tfrac{1}{2}m^2\phi'^2-\phi'^4). \tag{1.3}$$

Thus, $g$ does not appear in the field equations; if one can solve the theory for any positive $g$, one can solve it for any other positive $g$; $g$ is irrelevant. Another way of seeing the same thing is to observe that, in classical physics, $g$ is a dimensionful parameter and can always be scaled to one.

Of course, $g$ *is* relevant in quantum physics. The reason is that quantum physics contain a new constant, $\hbar$, and the important object (for example, in Feynman's path-integral formula) is

$$\frac{\mathscr{L}}{\hbar}=\frac{1}{g^2\hbar}(\tfrac{1}{2}\partial_\mu\phi'\partial^\mu\phi'+\ldots). \tag{1.4}$$

As we see from this expression, the relevant (dimensionless) parameter is $g^2\hbar$, and thus semiclassical approximations, small-$\hbar$ approximations, are tantamount to weak-coupling approximations, small-$g$ approximations.

At this point you must be puzzled by the trumpets and banners of my opening paragraph. Do we not have a perfectly adequate small-coupling approximation in perturbation theory? No, we do not; there is a host of interesting phenomena which occur for small coupling constant and for which perturbation theory is inadequate.

The easiest way to see this is to descend from field theory to particle mechanics. Consider the theory of a particle of unit mass moving in a one-dimensional potential,

$$L=\tfrac{1}{2}\dot{x}^2-V(x;g), \tag{1.5}$$

where

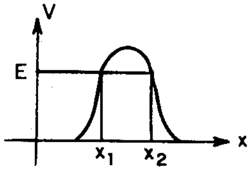
$$V(x;g)=\frac{1}{g^2}F(gx), \tag{1.6}$$

and $F$ is some function whose Taylor expansion begins with terms of order $x^2$. Everything I have said about the field theory defined by Eq. (1.1) goes through for this theory. However, let us consider the phenomenon of transmission through a potential barrier (Fig. 1). Every child knows that the amplitude for transmission obeys the WKB formula,

$$|T(E)|=\exp\left\{-\frac{1}{\hbar}\int_{x_1}^{x_2}\mathrm{d}x[2(V-E)]^{\frac{1}{2}}\right\}[1+O(\hbar)], \tag{1.7}$$

where $x_1$ and $x_2$ are the classical turning points at energy $E$. This is a semiclassical approximation. Nevertheless, transmission, barrier penetra-

**Fig. 1**



tion, is not seen in any order of perturbation theory, because Eq. (1.7) vanishes more rapidly than any power of $\hbar$, and therefore of $g$.

I can now make my first paragraph more explicit. There are phenomena in quantum field theory, and in particular in quantum chromodynamics, analogous to barrier penetration in quantum particle mechanics. In the last two years a method has been developed for handling these phenomena. This method is the subject of these lectures.

The organization of these lectures is as follows. In Sect. 2 I describe the new method in the context of particle mechanics, where we already know the answer by an old method (the WKB approximation). Here the instantons which play a central role in the new method and which have given these lectures their title first appear. In Sect. 3 I derive some interesting properties of gauge field theories. In Sect. 4 I discuss a two-dimensional model in which instantons lead to something like quark confinement and explain why a similar mechanism has (unfortunately) no chance of working in four dimensions. In Sect. 5 I explain 't Hooft's resolution of the U(1) problem. In Sect. 6 I apply instanton methods to vacuum decay. Only this last section reports on my own research; all the rest is the work of other hands.[1]

I thank C. Callan, R. Dashen, D. Gross, R. Jackiw, M. Peskin, C. Rebbi, G. 't Hooft, and E. Witten for patiently explaining large portions of this subject to me. Although I have never met A. M. Polyakov, his influence pervades these lectures, as it does the whole subject.[2]

***A note on notation.*** In these lectures we will work in both Minkowski space and in four-dimensional Euclidean space. A point in Minkowski space is labeled $x^\mu$, where $\mu=0, 1, 2, 3$, and $x^0$ is the time coordinate. In Minkowski space I will distinguish between covariant and contravariant vectors, $x_\mu = g_{\mu\nu}x^\nu$, where the metric tensor has signature $(+---)$. Euclidean space is obtained from Minkowski space by formal analytic continuation in the time coordinate, $x^4 = -\mathrm{i}x^0$. A point in Euclidean space is labeled $x^\mu$, where $\mu = 1, 2, 3, 4$. The signature of the metric tensor is $(++++)$. Thus covariant and contravariant vectors are component-by-component identical, and I will not bother to distinguish between them. Note that $x \cdot y$ in Minkowski space continues to $-x \cdot y$ in Euclidean space. The Euclidean action is defined as $-\mathrm{i}$ times the continuation of the Minkowskian action. When discussing particle problems, I will use $t$ for both Euclidean and Minkowskian time; which is meant will always be clear from the context. In Sect. 2 explicit factors of $\hbar$ are retained; elsewhere, $\hbar$ is set equal to one.

I have chosen it because it is familiar and concrete, but in some ways it is a bad choice for our purposes. Firstly, the model involves, not one scalar field, but many scalar and vector fields. Secondly, the vacuum stability features I have described are not properties of the classical potential, $U(\phi)$, but require consideration of one-loop corrections. Thus the formalism I am going to develop is not applicable to this case. As long as we are talking about this model, though, you might be tempted to consider the possibility that the Higgs mass is less than Weinberg's lower bound, that we are living in the false vacuum. As Linde[41] has pointed out, this is silly; if this were the case, there would be no way for the universe to get into the false vacuum in the first place.)

The relevant parameter for cosmology is that cosmic time for which the product of $\Gamma/V$ and the volume of the past light cone is of order unity. If this time is on the order of microseconds, the universe is still hot when the false vacuum decays, even on the scale of high-energy physics, and a zero-temperature computation of $\Gamma/V$ is inapplicable. If this time is on the order of years, the decay of the false vacuum will lead to a sort of secondary big bang, with interesting cosmological consequences. If this time is on the order of billions of years, we have occasion for anxiety.

### 6.2 *The bounce*

We know from Sect. 2.4 how to compute $\Gamma/V$. We must find the bounce, $\bar{\phi}$, a solution of the Euclidean equations of motion,

$$\partial_\mu \partial_\mu \bar{\phi} = U'(\bar{\phi}), \tag{6.2}$$

that goes from the false ground state at time minus infinity to the false ground state at time plus infinity,

$$\lim_{x_4 \to \pm\infty} \bar{\phi}(\mathbf{x}, x_4) = \phi_+. \tag{6.3}$$

To these boundary conditions we can add another. It is easy to see that if the action of the bounce is to be finite,

$$\lim_{|\mathbf{x}| \to \infty} \bar{\phi}(\mathbf{x}, x_4) = \phi_+. \tag{6.4}$$

Once we have found the bounce, it is trivial to compute $\Gamma/V$. To leading order in $\hbar$,

$$\Gamma/V = K e^{-S_0}, \tag{6.5}$$

where $S_0$ is $S(\bar{\phi})$ and $K$ is a determinantal factor, defined as in Sect. 2.4.

I will shortly construct the bounce. Before I do so, though, I want to make some comments:

(1) We already see the power of our method. The problem of barrier penetration in a system with an infinite number of degrees of freedom has

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**Sidney Coleman's lectures on QFT**
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**Tony Zee's lectures on QFT**
**http://www.asti.ac.za/lectures.php**

**BOOKS**

**Quantum Field Theory in a Nutshell (Anthony Zee)**

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**WEBSITES**

**Quantum Field Theory in a Nutshell on the Web**
**http://press.princeton.edu/titles/7573.html**
**http://www.kitp.ucsb.edu/~zee/QuantumFieldTh.html**
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